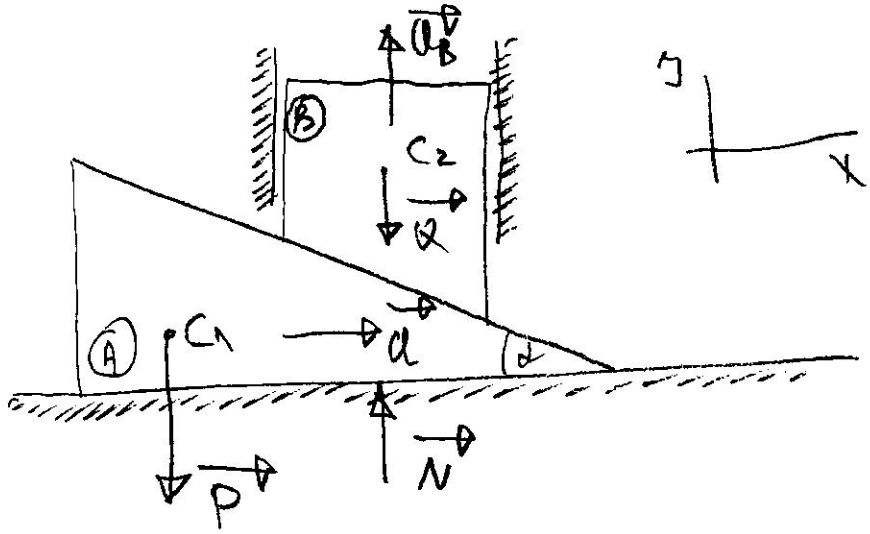
8.1

A: $P, a$

B: $Q$

$\vec{a}_c - ?$




$n = 2$

$m_1 = \frac{P}{g}, m_2 = \frac{Q}{g}$

$M = m_1 + m_2 = \frac{1}{g}(P + Q)$

$tg\alpha = \frac{y}{x} \Rightarrow y = x\, tg\alpha \; \Big| \frac{d^2}{dt^2}$

$\ddot{y} = \ddot{x}\, tg\alpha$

$a_B = a\, tg\alpha$

$M\vec{a}_c = \sum m_i \vec{a}_i$

$\frac{P+Q}{g}\vec{a}_c = \frac{P}{g}a\vec{i} + \frac{Q}{g}a\, tg\alpha\, \vec{j}$

$\vec{a}_c = \frac{a}{P+Q}\left(P\vec{i} + Q\, tg\alpha\, \vec{j}\right)$

8.2

$A: m_1$

$B: m_2$

$\overline{AB} = l$

$t_0 = 0 \quad \varphi(t_0) = \varphi_0 \ , \ \dot{\varphi}(t_0) = 0$

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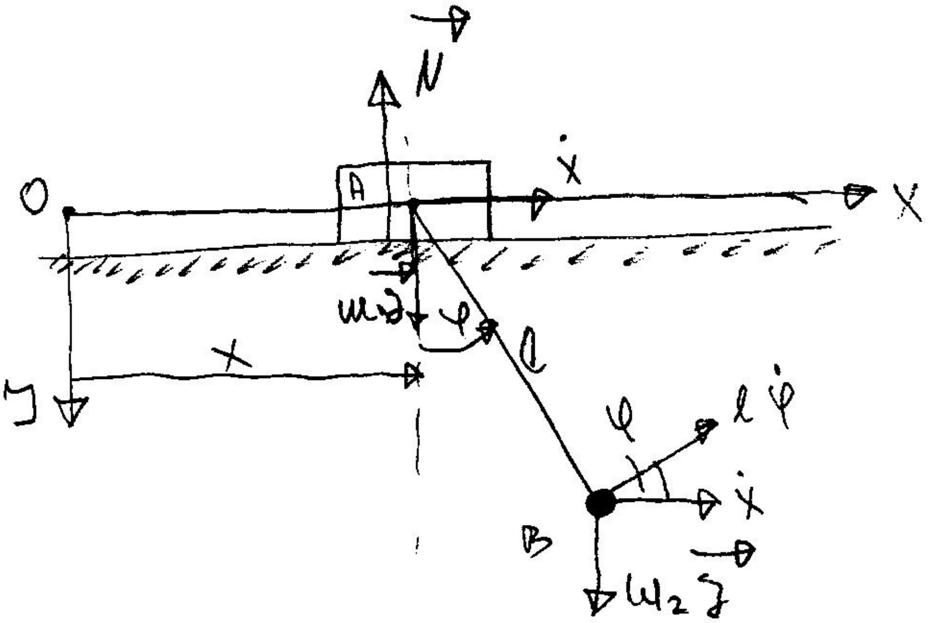
a) $x_C$ - ?

b) $x_B$ - ?




$$M \vec{a}_C = \vec{F}^e_R = m_1 \vec{g} + m_2 \vec{g} + \vec{N}$$

$$M X_C = \sum_i m_i x_i = \sum m_i x_{i0}$$

$$x: \ M \ddot{X}_C = 0 \ , \ M = m_1 + m_2$$

$$K_x = K_{x0} = const. \ , \ K_{x0} = m_1 \dot{x}_0 + m_2 (\dot{x}_0 + l \dot{\varphi}_0) = 0$$

$$\Rightarrow (m_1 + m_2) X_C = m_1 x + m_2 (x + l \sin\varphi) =$$

$$= m_1 \underbrace{x_0}_{=0} + m_2 (\underbrace{x_0}_{=0} + l \sin\varphi_0) = m_2 l \sin\varphi_0$$

$$\boxed{X_C = \frac{m_2 l \sin\varphi_0}{m_1 + m_2}}$$

$$(m_1+m_2)x + m_2 l\sin\varphi = m_2 l\sin\varphi_0$$

$$x = \frac{m_2 l(\sin\varphi_0 - \sin\varphi)}{m_1+m_2}$$

$$x_B = x + l\sin\varphi \quad , \quad y_B = l\cos\varphi$$

$$x_B = \frac{m_2 l(\sin\varphi_0 - \sin\varphi)}{m_1+m_2} + l\sin\varphi =$$

$$= \frac{m_2 l\sin\varphi_0 - \cancel{m_2 l\sin\varphi} + m_1 l\sin\varphi + \cancel{m_2 l\sin\varphi}}{m_1+m_2} =$$

$$= \frac{l(m_2\sin\varphi_0 + m_1\sin\varphi)}{m_1+m_2}$$

$$x_B(m_1+m_2) = l m_2\sin\varphi_0 + l m_1\sin\varphi \quad / : m_1$$

$$x_B\frac{m_1+m_2}{m_1} - \frac{m_2}{m_1} l\sin\varphi_0 = l\sin\varphi$$

$$\left.\begin{aligned} \frac{m_1+m_2}{m_1}\left(x_B - \frac{m_2}{m_1+m_2} l\sin\varphi_0\right) &= l\sin\varphi \quad /^2 \\ y_B &= l\cos\varphi \quad /^2 \end{aligned}\right\} +$$

$$\frac{\left(x_B - \frac{m_2}{m_1+m_2} l\sin\varphi_0\right)^2}{\left(\frac{m_1 l}{m_1+m_2}\right)^2} + \frac{y_B^2}{l^2} = 1 \text{ - ЕЛИПСА}$$

8.3

$$X = \frac{m_2 l(\sin\pi - \sin\varphi)}{m_1+m_2} = -\frac{m_2}{m_1+m_2}\, l \sin\varphi$$

$$\dot X = -\frac{m_2}{m_1+m_2}\, l\dot\varphi\cos\varphi = 0 \Rightarrow \varphi^* = \frac{\pi}{2}$$

$$X_1 = X\left(\varphi^*=\frac{\pi}{2}\right) = -\frac{m_2}{m_1+m_2}\, l \qquad \varphi\left[\pi, \frac{\pi}{2}\right]$$

$$X_2 = X(\varphi=0) = 0 \qquad \varphi\left[\frac{\pi}{2}, 0\right]$$

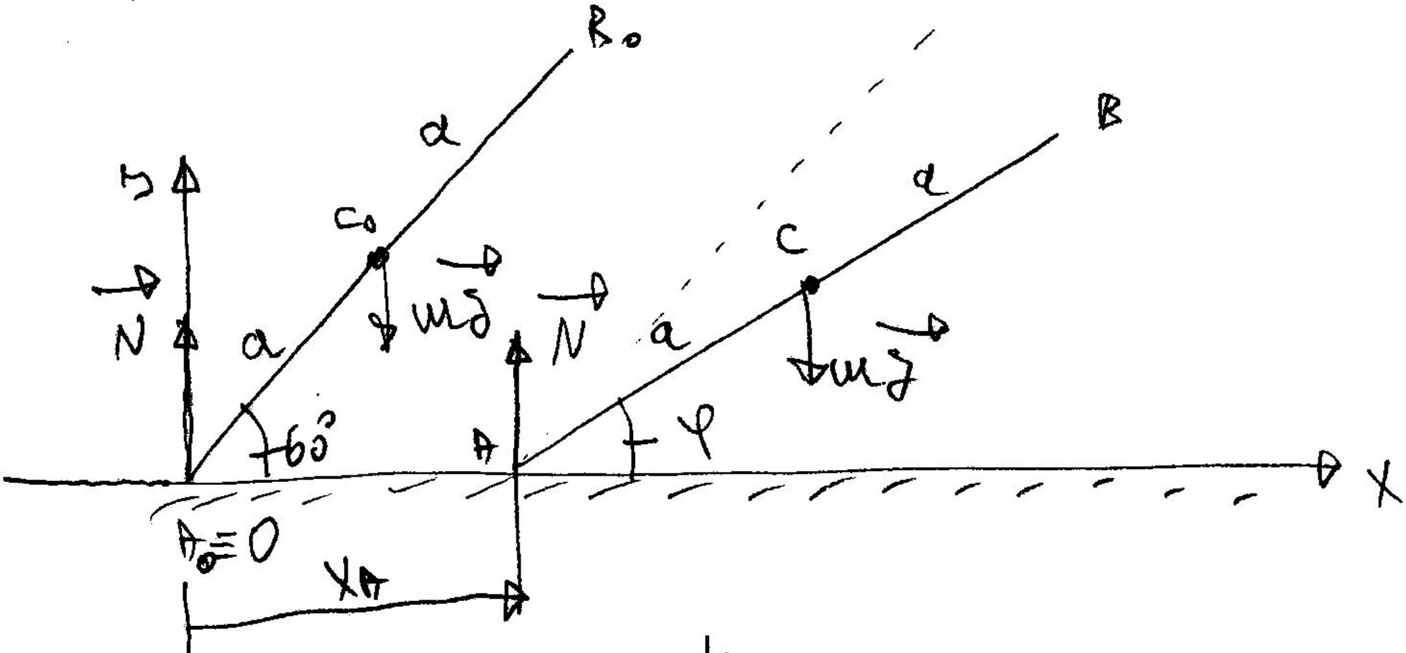
$$\boxed{\Delta X} = |X_1 - \underbrace{X_0}_{=0}| + |X_2 - X_1| = \frac{m_2}{m_1+m_2}\, l + \frac{m_2}{m_1+m_2}\, l = \boxed{\frac{2m_2}{m_1+m_2}\, l}$$

8.4 $\overline{AB} = 2a$

$t_0 = 0$ $\varphi_0 = 60^\circ$, МИРОВАО

a) $X_A = X_A(\varphi)$

б) $X_B, y_B$



$$K_x = K_{x0} = 0$$

$$M X_c = const. = M X_{c0}$$

$$m X_c = ma \cos 60^\circ = \frac{1}{2} m a$$

$$X_c = X_A + a \cos\varphi$$

$$x_A + a\cos\varphi = \frac{1}{2} a$$

$$x_A = \frac{a}{2} - a\cos\varphi = a\left(\frac{1}{2} - \cos\varphi\right)$$

---

$$x_B = x_A + 2a\cos\varphi = a\left(\frac{1}{2} + \cos\varphi\right)$$

$$y_B = 2a\sin\varphi$$

---

$$\left.\begin{aligned} \frac{1}{a}\left(x_B - \frac{a}{2}\right) &= \cos\varphi \quad )^2 \\ \frac{y_B}{2a} &= \sin\varphi \quad )^2 \end{aligned}\right\} +$$

---

$$\frac{\left(x_B - \frac{a}{2}\right)^2}{a^2} + \frac{y_B}{(2a)^2} = 1 \text{ - ЕЛИПСА}$$

---

8.5

$A: m_1$

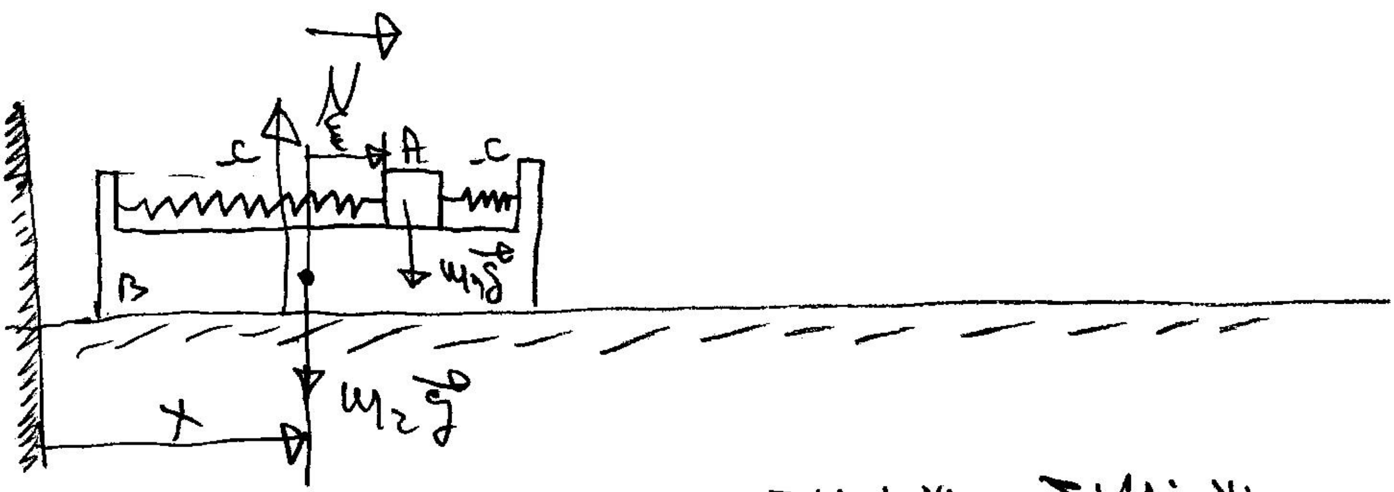
$B: m_2$

$c$

$t_0=0 \quad x(t_0)=x_0,\ \dot{x}(t_0)=0$

---

$x_B(t) - ?$

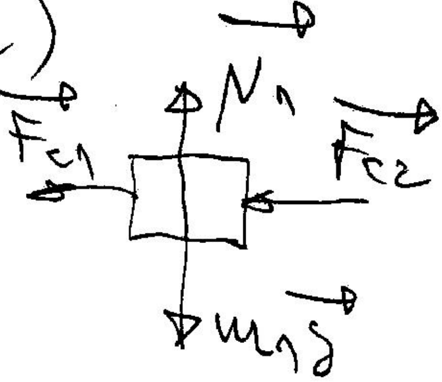



$$K_x = K_{x0} = 0, \quad M x_C = \sum_i m_i x_i = \sum_i m_i x_{i0}$$

$$M x_C = m_1(\underline{x}+\xi) + m_2 \underline{x} = m_1 x_0$$

$$x(m_1+m_2) + m_1 \xi = m_1 x_0$$

$$x = \frac{m_1}{m_1+m_2}(x_0 - \xi)$$




ТЕЛО А:

$$m_1(\ddot{x}+\ddot{\xi}) = -2c\xi$$

$$\ddot{x} = -\frac{m_1}{m_1+m_2}\ddot{\xi}$$

$$m_1\left(\ddot{\xi}-\frac{m_1}{m_1+m_2}\ddot{\xi}\right)=-2c\xi$$

$$\frac{m_1 m_2}{m_1+m_2}\ddot{\xi}=-2c\xi \quad \Big| \cdot \frac{m_1+m_2}{m_1 m_2}$$

$$\ddot{\xi}+2c\frac{m_1+m_2}{m_1 m_2}\xi=0, \quad \ddot{\xi}+w^2\xi=0$$

$$w^2=2c\frac{m_1+m_2}{m_1 m_2}$$

$$\xi=Ae^{\lambda t}$$

$$\dot{\xi}=A\lambda e^{\lambda t}$$

$$\ddot{\xi}=A\lambda^2 e^{\lambda t}$$

$$A\lambda^2 e^{\lambda t}+w^2 A e^{\lambda t}=0$$

$$(\lambda^2+w^2)Ae^{\lambda t}=0$$

$$\lambda^2+w^2=0 \Rightarrow \lambda=\pm iw$$

$$\xi=A_1 e^{iwt}+A_2 e^{-iwt}$$

$$e^{\pm iwt}=\cos wt \pm i\sin wt$$

$$\xi=A_1(\cos wt+i\sin wt)+A_2(\cos wt-i\sin wt)$$

$$\xi=(A_1+A_2)\cos wt+i(A_1-A_2)\sin wt$$

$$A_1+A_2=C_1 \;;\; i(A_1-A_2)=C_2$$

$$\xi=C_1\cos wt+C_2\sin wt$$

$$\dot{\xi} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$$

---

$t_0 = 0 \quad \xi(t_0) = X_0, \quad \dot{\xi}(t_0) = 0$

$X_0 = C_1$

$0 = C_2 \omega \Rightarrow \boxed{C_2 = 0}$

$$\xi = X_0 \cos \omega t = X_0 \cos \sqrt{2C \frac{m_1 + m_2}{m_1 m_2}}\, t$$

$$X = \frac{m_1 X_0}{m_1 + m_2} \left( 1 - \cos \sqrt{2C \frac{m_1 + m_2}{m_1 m_2}}\, t \right)$$

---

8\. $g$ $n$: $m_1$

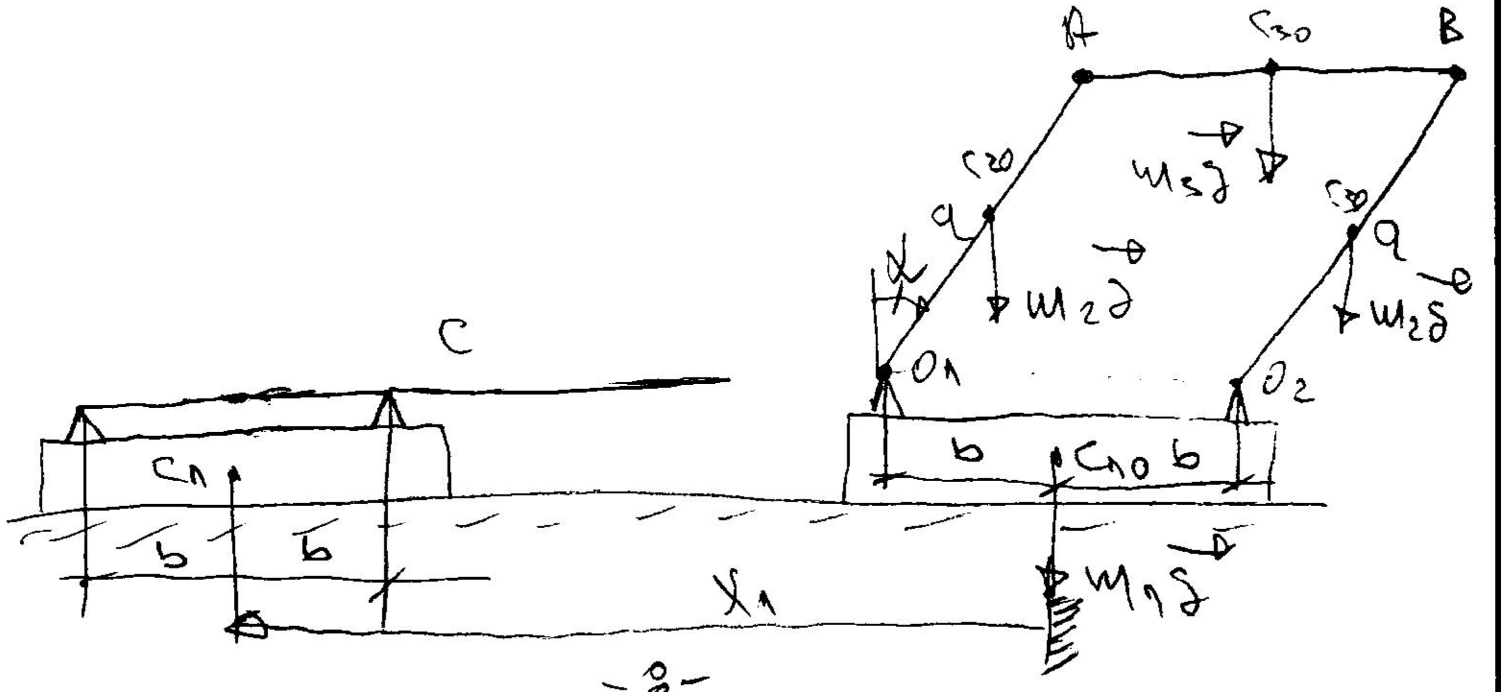
$\overline{O_1 A} = \overline{O_2 B} = a$, $m_2$

$AB$: $m_3$

$t_0 = 0 \quad \varphi_0 = \alpha$

---

$\Delta X_1$



$$V_x = V_{x0} = 0$$

$$M x_c = \sum_i m_i x_i = \sum_i m_i x_{i0}$$

$$M = m_1 + 2m_2 + m_3$$

$$(m_1 + 2m_2 + m_3) x_c = m_1 \underline{x_1} + m_2\left(\underline{x_1} + \cancel{b} - \frac{a}{2}\right) +$$

$$+ m_2\left(\underline{x_1} - \cancel{b} - \frac{a}{2}\right) + m_3\left(\underline{x_1} + b - a - \frac{\overline{AB}}{2}\right) =$$

$$= 0 + m_2\left(\cancel{b} - \frac{a}{2}\sin\alpha\right) + m_2\left(-\cancel{b} - \frac{a}{2}\sin\alpha\right) +$$

$$+ m_3\left(b - a\sin\alpha - \frac{\overline{AB}}{2}\right)$$

$$x_1 (m_1 + 2m_2 + m_3) - m_2 a + \cancel{m_3 b} - m_3 a - \cancel{m_3 \frac{\overline{AB}}{2}} =$$

$$= -a m_2 \sin\alpha + \cancel{m_3 b} - m_3 a \sin\alpha - \cancel{m_3 \frac{\overline{AB}}{2}}$$

$$x_1 (m_1 + 2m_2 + m_3) = m_2 a + m_3 a - m_2 a \sin\alpha -$$

$$- m_3 a \sin\alpha$$

$$x_1 = a\,\frac{m_2(1-\sin\alpha) + m_3(1-\sin\alpha)}{m_1 + 2m_2 + m_3} =$$

$$= a(1-\sin\alpha)\,\frac{m_2 + m_3}{m_1 + 2m_2 + m_3}$$

8.14 A: $G_3$, r

$3r$, $\omega$

$\overline{OA} = 2r$, $G_2$

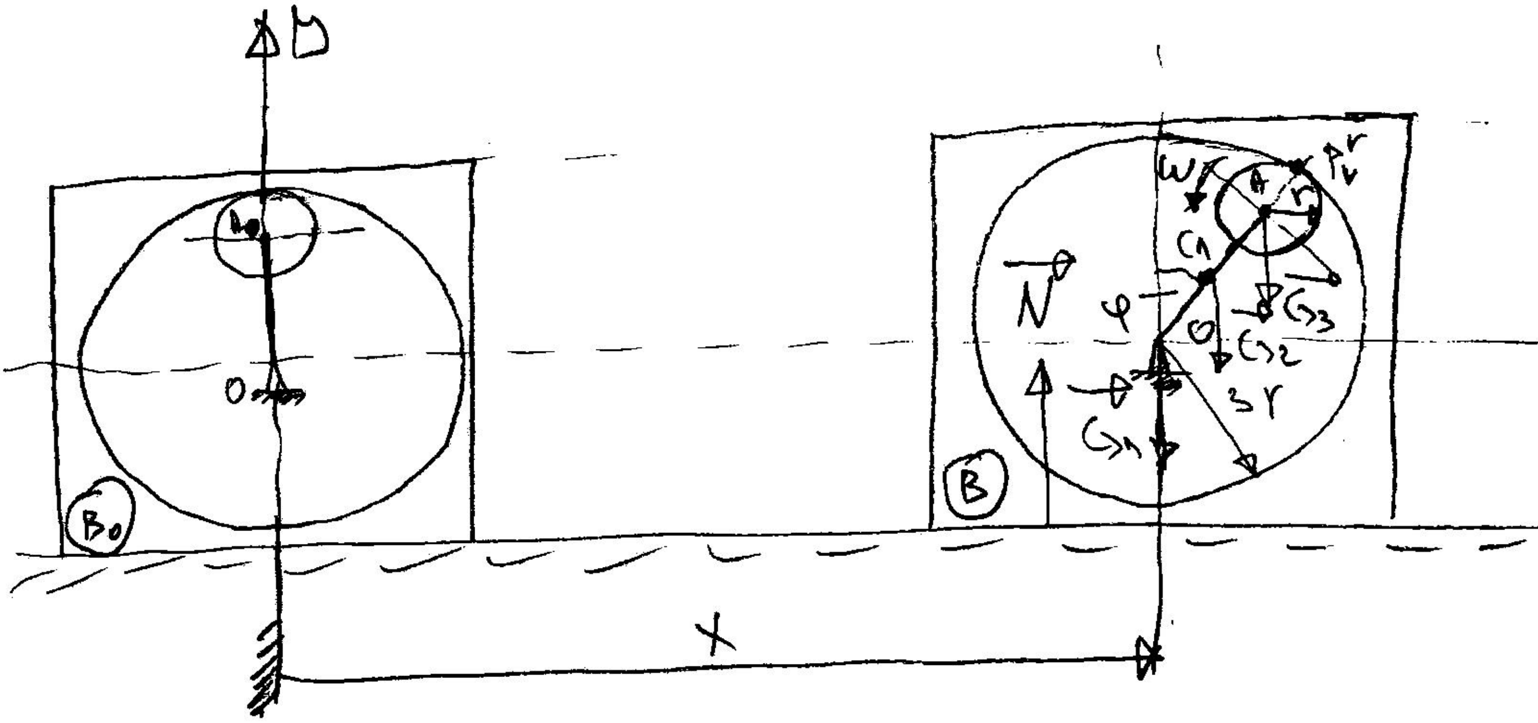
B: $G_1$

$t_0 = 0$ $\varphi_0 = 0$

a) $x$ - ?

b) $N$ - ?

c) $\omega_{max}$ - ?




$M = \frac{1}{g}(G_1 + G_2 + G_3)$

$x_B = x$

$K_x = K_{x0} = const.$

$M \dot{x}_C = M \dot{x}_{C0}$

$x_O = x$

$x_{C1} = x + r\sin\varphi$

$x_A = x + 2r\sin\varphi$

$y_O = 0$

$y_{C1} = r\cos\varphi$

$y_A = 2r\cos\varphi$

$$\frac{1}{g}G_1\dot{x} + \frac{1}{g}G_2(\dot{x} + r\dot{\varphi}\cos\varphi) + \frac{1}{g}G_3(\dot{x} + 2r\dot{\varphi}\cos\varphi) =$$

$$= 0 + \frac{1}{g}G_2 r\dot{\varphi}_0\cos\varphi_0 + \frac{1}{g}G_3 w\dot{\varphi}_0\cos\varphi_0 \quad \Big|\, g$$

$$2\cancel{r}\dot{\varphi} = \cancel{r}w \Rightarrow \dot{\varphi} = \frac{w}{2} \Rightarrow \varphi = \underbrace{\varphi_0}_{=0} + \frac{w}{2}t$$

$$\boxed{\varphi = \frac{w}{2}t}$$

$$\dot{x}(G_1+G_2+G_3) + r\frac{w}{2}\cos\frac{w}{2}t\,(G_2+2G_3) =$$

$$= r\frac{w}{2}(G_2+2G_3)$$

$$\dot{x} = \frac{(G_2+2G_3)(1-\cos\frac{w}{2}t)}{G_1+G_2+G_3}\,r\frac{w}{2} = \frac{dx}{dt}$$

$$dx = r\frac{w}{2}\,\frac{G_2+2G_3}{G_1+G_2+G_3}\left(1-\cos\frac{w}{2}t\right)dt \quad \Big|\int$$

$$x = \underbrace{x_0}_{=0} + \frac{rw}{2}\,\frac{G_2+2G_3}{G_1+G_2+G_3}\left(t - \frac{2}{w}\sin\frac{w}{2}t\,\Big|_0^t\right)$$

$$\boxed{x = \frac{G_2+2G_3}{G_1+G_2+G_3}\left(\frac{rw}{2}t - r\sin\frac{wt}{2}\right)}$$

$$M\vec{a}_c = \sum m_i \vec{a}_i = \vec{F}_R^s$$

$$y:\quad M\ddot{y}_c = \frac{1}{g}G_1\cdot 0 + \frac{1}{g}G_2\,\ddot{y}_{c1} + \frac{1}{g}G_3\,\ddot{y}_A =$$

$$= N - G_1 - G_2 - G_3$$

$$N = G_1 + G_2 + G_3 + \frac{1}{g}G_2\,\ddot{y}_{c1} + \frac{1}{g}G_3\,\ddot{y}_A \qquad (*)$$

$$\dot{y}_{c1} = -\dot{\varphi} r \sin\varphi \Rightarrow \ddot{y}_{c1} = -\dot{\varphi}^2 r \cos\varphi$$

$$\dot{y}_A = -2r\dot{\varphi}\sin\varphi \Rightarrow \ddot{y}_A = -2\dot{\varphi}^2 r\cos\varphi$$

$(*)$

$$N = (G_1 + G_2 + G_3) - \frac{1}{g}(G_2 + 2G_3)\frac{\omega^2}{4} r \cos\frac{\omega t}{2}$$

$$\Rightarrow \quad N_{min} \geq 0$$

$$(G_1 + G_2 + G_3) \geq \frac{1}{g}(G_2 + 2G_3)\frac{\omega^2 r}{4}$$

$$\omega \leq \sqrt{\frac{4g(G_1 + G_2 + G_3)}{(G_2 + 2G_3)\, r}}$$

$$\boxed{\omega_{max} = \sqrt{\frac{4g}{r}\,\frac{G_1 + G_2 + G_3}{G_2 + 2G_3}}}$$